MODEL THEORY 2024

EXAM FAC SIMILE

Exercise 1.
- Define the set of (complete) (1)-types of a theory.
- For a theory $\mathbb{T}$ what does it mean to admit quantifiers elimination?
- Define elementary embeddings.

Exercise 2. Recall that a theory is $\aleph_0$-categorical if it has only one model of cardinality $\aleph_0$ up to isomorphism. List at least two $\aleph_0$-categorical theories (over some finite first order language). Is the theory of algebraically closed fields $\aleph_0$-categorical?

Exercise 3. Consider two theories $\mathbb{T} \subset \mathbb{T}'$ over the same language, and assume that $\mathbb{T}'$ extends $\mathbb{T}$. Every n-type over $\mathbb{T}'$ is also an n-type over $\mathbb{T}$. This gives us a function between spaces of types $\mathrm{tp}^n(\mathbb{T}') \to \mathrm{tp}^n(\mathbb{T})$. We say that $\mathbb{T}'$ is a conservative extension if every formula provable in $\mathbb{T}'$ is also provable in $\mathbb{T}$.
Show that this function is continuous and prove that if $\mathbb{T}'$ is a conservative extension of $\mathbb{T}$ then the function is surjective.

Exercise 4. Let us start with a brief definition. We say that an element b of M is *algebraic* over a subset $A \subset M$ if b belongs to a finite A-definable set, i.e. there exists a formula $\phi(x)$ (with paramenter in A) such that $\phi(x)$ has a finite number of solutions in M and b is one of them. For a set A we define its algebraic closure $\mathrm{acl}(A)$ to be the set of elements that are algebraic over A.
Show that if M is an ω-saturated model, then there exists no finite subset A whose algebraic closure is the whole M.

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